The Right Stuff: Appropriate Mathematics for All Students

Promoting the use of materials that engage students in meaningful activities that promote the effective use of technology to support mathematics, further equip students with stronger problem solving and critical thinking skills, and enhance numeracy.

Overview

Students will apply the concepts of

- Numeracy – Students will examine data numerically and graphically and be able to describe their observations in mathematical terms.
- Modeling – Students will model data with a polynomial function using technology.
- Rate of Change – Students will calculate the average rate of change between two MPH and be able to explain how the instantaneous rate of change (found with technology) changes; especially at the inflection point.
- Quantitative Literacy – Students will calculate the time and the cost of a trip.

Supplies and Materials

- 1.1 Student Worksheet
- Either 1.3 Excel file, 1.4 TI-Nspire™ file, or a handheld that will create a scatter plot and find a model for the data

Prerequisite Knowledge

Students must be able copy data from word into Excel or into a handheld, create a scatter plot, and find an appropriate model for the data.

Instructional Suggestions

1. Have students discuss the graph in the introduction and compare their experiences with what the graph suggests.
2. Examine the raw data. What should be done to make the data easier to analyze? Ask students to guess the shape of the curve the data seems to make.
3. Find an appropriate model to fit the data. Discuss the degree of accuracy you expect from that model. Why would it not be easy to predict the correct MPG for any MPH? What size error would you expect?
4. Discuss the different models students obtain and especially examine the coefficient of the squared term. Discuss the number of decimals required.
5. The curve decreases at an increasing rate and then decreases at a decreasing rate. Find the MPH at which that change happens. Discuss the change in the MPG per MPH near that point.

Assessment Ideas

Have students find a polynomial model for the data shown in Figure 1 representing the distance traveled over time. Also have them find the average speed for the first hour, the second hour, and the entire trip. Discuss why the averages might be different. If a highway patrolman clocked the vehicle at 10:06, what do you think their speed was? at 11:06?

time	distance traveled
10:00	0
10:13	13.5
10:25	28
10:46	49.5
11:03	63
11:11	69
11:22	78
11:31	87
11:42	99
11:54	113
12:00	124

Figure 1

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Introduction

The U S Government offers drivers tips to save money by driving more efficiently. The tips include driving more sensibly, removing excess weight, using cruise control, using overdrive gears, and obeying the speed limit.
http://www.fueleconomy.gov/feg/driveHabits.shtml

The speed at which you drive affects your fuel economy. While each vehicle reaches its optimal fuel economy at different speeds, gas mileage usually decreases rapidly at speeds above 60 miles per hour (mph).

Figure 2 was taken from the government website mentioned above and indicates estimates for fuel economy (mpg) versus speed (mph).

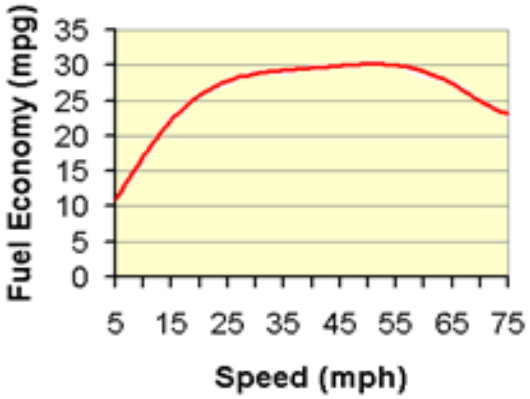

Figure 2

Discuss this graph. Is this data true for all cars? for all drivers? for all regions of the country? How do you think the data was calculated? At what speed is the fuel economy the best? worst?

Directions

1. Consider the following scenario:

Ma and Pa routinely make a 77 mile trip down I-40 to see their grandchildren. Since the trip includes 66 miles of non-stop travel, they can choose to travel at different speeds. The speed limit on this stretch of road is 70 mph. While they drive the 66-mile stretch, they decide to experiment with their speed and watch the gas mileage since their vehicle has the ability to calculate the miles per gallon over any specified trip as well as show the average speed.

The data they collected over 27 trips is shown here. From the data, you are to describe the change in the MPH (miles per hour) per MPG (miles per gallon).

What would make the data in this table easier to analyze?

If the data was sorted by MPH it would be easier to read.
Ask which variable the driver controlled: the MPH or the MPG. This should help the student understand why the MPH is the independent variable.
Ask students if they can find optimal values. Finally, is there a difference between Ma and Pa's driving habits?

Trip #	Driver	Avg MPH	Avg MPG
1	Pa	**66**	**26.6**
2	Ma	**57**	**29.4**
3	Pa	**80**	**18.5**
4	Ma	**57**	**29.5**
5	Ma	**62**	**28.8**
6	Pa	**73**	**20.5**
7	Pa	**71**	**21.6**
8	Ma	**63**	**28.2**
9	Pa	**60**	**29.2**
10	Ma	**56**	**29.4**
11	Ma	**61**	**29**
12	Pa	**68**	**25**
13	Pa	**65**	**27**
14	Pa	**65**	**26.9**
15	Ma	**59**	**29.5**
16	Ma	**58**	**29.6**
17	Ma	**75**	**19.5**
18	Ma	**58**	**29.5**
19	Pa	**70**	**22.8**
20	Pa	**68**	**25.2**
21	Ma	**56**	**29.4**
22	Ma	**55**	**29.3**
23	Pa	**67**	**25.8**
24	Pa	**70**	**22.7**
25	Ma	**55**	**29.2**
26	Pa	**67**	**25.7**
27	Pa	**76**	**19.2**

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2. Sort the data by Column C: "Avg MPH."

Describe any trends you see.

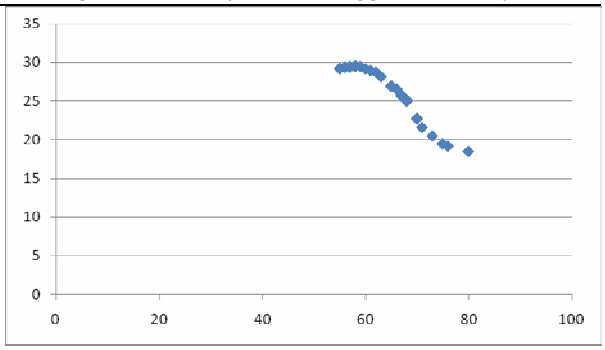
Pa drives faster than Ma.
As the MPH increases beyond 60, the MPG decreases.
There appears to be a maximum MPG that occurs at about 58 MPH.

Trip #	Driver	Avg MPH	Avg MPG
3	Pa	**80**	**18.5**
27	Pa	**76**	**19.2**
17	Ma	**75**	**19.5**
6	Pa	**73**	**20.5**
7	Pa	**71**	**21.6**
19	Pa	**70**	**22.8**
24	Pa	**70**	**22.7**
12	Pa	**68**	**25**
20	Pa	**68**	**25.2**
23	Pa	**67**	**25.8**
26	Pa	**67**	**25.7**
1	Pa	**66**	**26.6**
13	Pa	**65**	**27**
14	Pa	**65**	**26.9**
8	Ma	**63**	**28.2**
5	Ma	**62**	**28.8**
11	Ma	**61**	**29**
9	Pa	**60**	**29.2**
15	Ma	**59**	**29.5**
16	Ma	**58**	**29.6**
18	Ma	**58**	**29.5**
2	Ma	**57**	**29.4**
4	Ma	**57**	**29.5**
10	Ma	**56**	**29.4**
21	Ma	**56**	**29.4**
22	Ma	**55**	**29.3**
25	Ma	**55**	**29.2**

3. Create a scatter plot of the data: "Avg MPH" versus "Avg MPG."

The default scale that Excel uses clusters the data too closely. What would you do to make the data more easily analyzed? What changes would you make to the domain and range of your graph?

By changing the scale, the data can be analyzed more easily.



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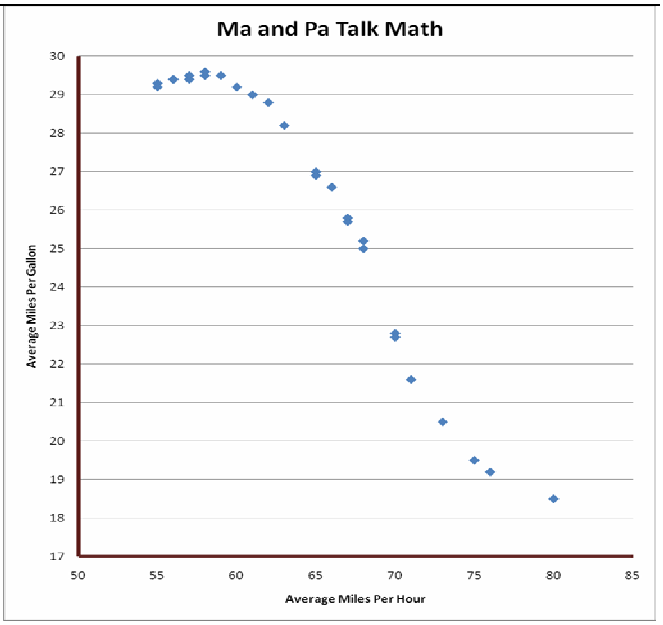
4. Change the scale on the horizontal axis (MPH) to a minimum of 50 and a maximum of 85.

   Change the scale of the vertical axis (MPG) to a minimum of 17 and a maximum of 30.

Ask them if all the points are shown on the graph? Some trips averaged the same MPH but slightly different MPG, can you find those trips? What if two or more trips had averaged the same MPH and MPG, could they be located on the graph? No. The points would be on top of one another. Is that a problem? This problem, in data mining, is real and is corrected by inserting "noise" on the y-variable by adding small random numbers to the x- and y-values.



5. Examine the data.
   a. Find the average change in MPG per MPH between trips 6 (73, 20.5) and 7 (71, 21.6).
   b. Find the average change in MPG per MPH between trips 13 (65, 27) and 14 (65, 26.9).
   c. Find the average change in MPG per MPH between trips 21 (56, 29.4) and 22 (55, 29.3).

By picking pairs of points, the data may be misleading or difficult to interpret. Thus, finding a function to model the data may make it helpful to compute the rate of change in MPG per MPH.

What function do you think might fit the data best?

It appears that a cubic polynomial function will fit the data well.

a. 73 20.5
71 21.6
The average change between these two trips is a decrease in 0.55 mpg per mph

b. 65 27
65 26.9
The average change between these two trips is undefined.

c. 56 29.4
55 29.3
The average change between these two trips is an increase of 0.1 mpg per mph

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6. Experiment with different models using the Trendline feature of Excel. Would you try linear? logarithmic? exponential? polynomial?

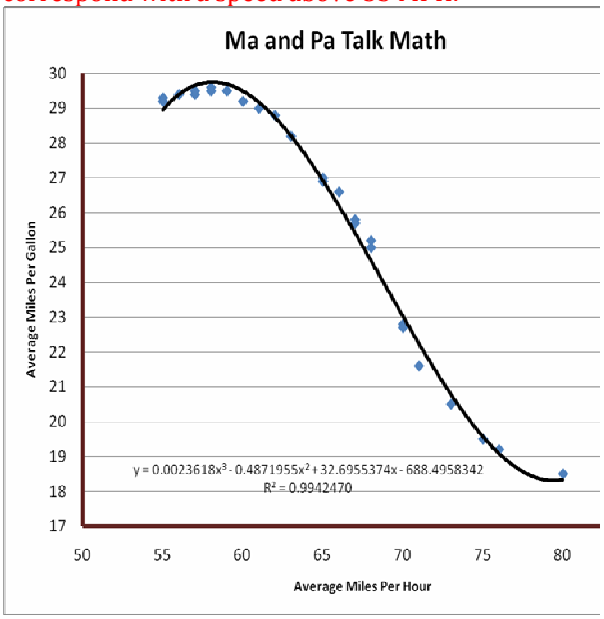
   Finally, after experimenting, create a new column next to the data with the heading "Model". Enter the formula for the cubic regression model, **y=0.0023618*C2^3-0.4871955*C2^2+32.6955*C2- 688.5** in this column. How does this compare to the model that you had chosen as best fit?

Compare the results of the model to the actual data. Is the model a fairly good one? Explain.

Compare your result to the graphic at right.

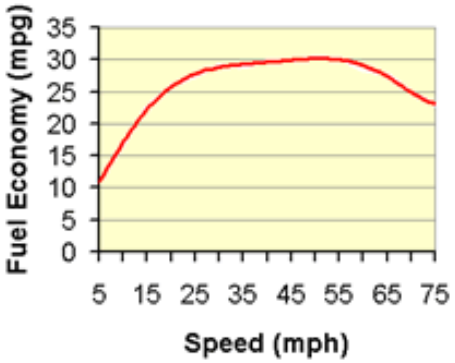
The model seems to fit the data reasonably well; it has the general shape even though at the speeds from 65 - 72 it does seem to be too "straight" and seems to begin to curl upward for speeds in excess of 80.

Note: One way we often deal with the problem of large data values used to create a model is to translate them. In this case, we could let 55 correspond with 0 and let the other speeds correspond with a speed above 55 MPH.



7. Compare the graph in the introduction to the graph in (6). Do the data that Ma and Pa obtained seem to follow the model the US Government advertises? Explain.

The graph in the introduction shows that as the speed goes beyond 60 the fuel efficiency decreases rapidly. This is validated by the data here. The maximum fuel efficiency for most cars appears to be between 55 and 60. That also appears in this data.



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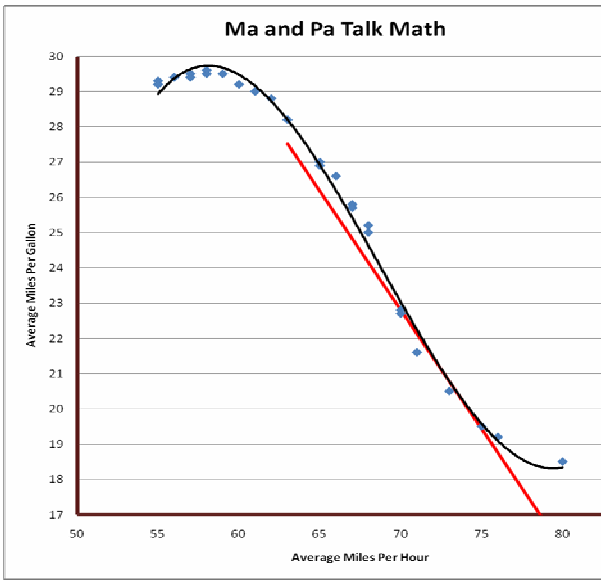
8. Analyze the graph.



At what speed does it appear that Ma and Pa get the best gas mileage?

Estimate the change in the gas mileage for every increase of 1 MPH in the speed between the speeds of

(a) 60 and 65 MPH.
(27-29.2)/5 = -0.46 MPG/MPH

(b) 65 and 70 MPH.
= -0.85 MPG/MPH

(c) 70 and 75 MPH.
= -0.65 MPG/MPH

To reinforce the concept of a change in the rate of change, use the moveable tangent line provided in the worksheet: "TANGENTLINE." <Using TI Nspire, place a point on the curve and a line tangent to the curve on the point. Show the slope of the line. Automatically collect data (x-coordinate, slope of tangent line) and show that set of points defines a parabola.> Ask the student about the slope of that tangent line as the model is increasing, near the max, decreasing at an increasing rate, and decreasing at a decreasing rate.

Be sure, when talking about the slope, that you use units. Slope is a ratio, and in this case the units of this ratio are MPG per MPH.

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9. Answer the following questions.

When Pa drives, he averages 69 miles per hour. When Ma drives, she averages 59 miles per hour. On the 66-mile stretch down I-40,

(a) How much *money* do they save (one way) when Ma drives rather than Pa, if the fuel cost is $1.99/gallon?

(b) How much *time* do they save (one way) when Pa drives rather than Ma?

It is important to continue to build the students' skills with numeracy. Questions like this require them to use dimensional (or unit) analysis to do arithmetic calculations to obtain reasonable solutions. Emphasize UNITS.

Ask students to consider the time saved versus the difference in MPH. Discuss methods used by students and what went wrong. How did they know there answer wasn't reasonable? Some may have divided 66 miles by 10 MPH. Did anyone use: (69/59)*(57.39 min) = 67.12 min ?

Discuss driving habits and how a driver's habits will influence their fuel efficiency.

Distance down I-40:					
66	miles				
$ 1.99	Fuel Cost per gallon				
(a)	Cost savings when Ma drives:			**$1.08**	
		avg speed	MPG	Gallons Used	Fuel Cost
	Pa	69	23.83	2.77	$ 5.51
	Ma	59	29.67	2.22	$ 4.43
(b)	Time saved when Pa drives:			**9.7 min.**	
		avg speed	time for trip (hrs)		Time in minutes
	Pa	69	0.956521739		57.39130435
	Ma	59	1.118644068		67.11864407

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