**The Right Stuff: Appropriate Mathematics for All Students**

***Promoting the use of materials that engage students in meaningful activities that promote the effective use of technology to support mathematics, further equip students with stronger problem solving and critical thinking skills, and enhance numeracy.***



**Overview**

Students will apply the concepts of

* Distance formula or the Pythagorean Theorem to write a function that describes the distance between a point and a line
* Linear, quadratic, piecewise, limits
* Modeling – Students will be able to find an appropriate model between two variables using technology
* Multiple Representations – Students will be able to represent the relationship between two variables in a table or graph

**Supplies and Materials**

* 10.1 Student Worksheet
* Either 10.3 Excel file, 10.4 TI-Nspire™ file, or a handheld that will create a scatter plot and find a model for the data

**Prerequisite Knowledge**

Students must be able to copy data from Word into Excel or into a handheld, create a scatter plot, and find an

appropriate algebraic model for the data. Students must be able to find the distance between two points using the

distance formula or Pythagorean Theorem.

**Instructional Suggestions**

This worksheet is designed to help students formulate a problem mathematically. The solution is intuitively obvious, but can be verified by developing an appropriate function and finding the minimum. This function can also be used to find when the car is a fixed distance from the cell tower. It is fairly straightforward for students to do the parts where the road is along the x- or y- axis, but roads that are diagonal are more challenging.   
The graph of the distance the vehicle is away from the tower at *t* = 600, 1200, 1800 and 2400 seconds may be approximated by a linear model.

**Assessment Ideas**

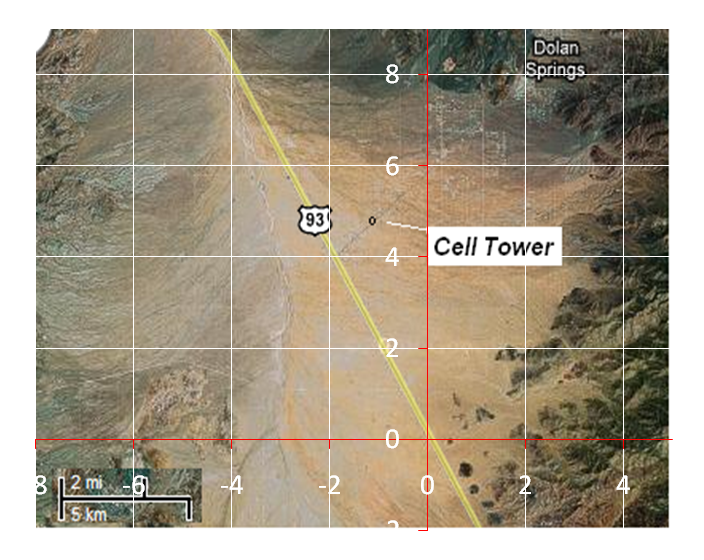
The student will demonstrate an ability to apply the Pythagorean Theorem or distance formula and determine a

function that describes the distance between a point and a line. The student may complete questions 6-14 or

15-22 as a cumulative assessment activity. As an extension, the student may be given an additional assignment

with a different cell tower position or road to travel.

**Introduction**



The picture to the right shows a section of US-93 near Kingman, Arizona. If a car is located at the point (0, 0) traveling at 60 miles per hour towards the cell tower along US-93, how long will it be before the signal from the tower is strongest? Assume that the signal is broadcast equally in all directions and its strength is inversely proportional to the distance.

FFigure 1

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| A vehicle, represented by the red dot at the point (0, 0) is traveling along a road represented by the *x-axis*. A cell tower is represented by the blue diamond at the point (1.5, 2). The scale is one unit equals one mile. The vehicle is traveling at a constant rate of 60 mph.   1. When do you think the vehicle will be a minimum from the cell tower and what is the minimum distance? Make a conjecture now.     By examining the coordinates of the point, a student should come up with the answer 1.5 minutes or 90 seconds and a minimum distance of 2 miles. Let them know that the next few steps will lead them to the solution mathematically. | The term “orthogonal” may be used when considering the relationship between the point and the line when the distance is minimized.  For formative assessment, change the speed to 30 mph and work individually. |
| 1. Use the Pythagorean Theorem to find the distance the car is from the cell tower at *t* = 0. | The Pythagorean Theorem should be a formula that is know by students. Having them draw the graph by hand and connecting the dots to form the right triangle should help them connect the concept and the formula.  D = √(1.52 + 22) = 2.5 miles |
| 1. Complete the table for other values of *t* using a constant speed of 60 mph.   The two given values should work as a check to insure that the students are doing the activity correctly. You may want to have a discussion about certain values matching and symmetry. | |  |  |  | | --- | --- | --- | | Time | Dist Car Travels x | Dist from Cell Tower | | (sec) | (miles) | (miles) | | **0** | **0** | **2.5** | | **30** | **0.5** | **2.236** | | **60** | **1** | **2.062** | | **90** | **1.5** | **2** | | **120** | **2** | **2.062** | | **150** | **2.5** | **2.236** | | **180** | **3** | **2.5** | | **210** | **3.5** | **2.828** | | **240** | **4** | **3.202** | |  |  |  | |
| 1. Complete the following:    1. Construct a scatter plot of the data, time vs Distance from the Cell Tower, and describe the shape.   By looking at the graph, students may recognize the symmetry discussed in (3) and also see that the shape of the graph is parabolic.   * 1. Consider the distance the vehicle is from the cell tower at *t* = 600,  *t* = 1200, *t* = 1800, and *t* = 2400. (Assume the vehicle continues to travel along the same path at the same speed).   A possible discussion with students would be what if time was a negative number. You could actually find the distance to the tower.   * 1. Describe the data you found in (b). Did the trend you saw in (a) continue? Explain.   The points that represent the time and distance from the cell tower for large values of t are no longer parabolic in shape. Rather, they appear almost linear.  Help students realize that the length of the longer leg of a right triangle approaches the length of the hypotenuse when the shorter leg is very small, or small relative to the longer leg. You could mention the concept of limit here. |  |

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| 1. You may have noticed that you can find the *x-value* of the point by dividing the time *t* by 60. This means that we can write the coordinates at any time as  (*t* / 60, 0). Find the formula for the distance between the point (*t* / 60, 0) and (1.5, 2). Solve for ***d***. | A good reminder for students is that the distance formula comes directly from the Pythagorean theorem. This may help them understand this formula: |
| 1. Graph the formula from (5) on the scatter plot from (4).   As an extension, you may have a student find the regression model to see how it compares to the formula from (5) and verify (either with algebra or by estimating) that the minimum of that curve is 90 seconds. |  |
| Now consider another scenario.  A vehicle, represented by the red dot at the point (0, 0) is traveling along a road represented by the *y-axis*. A cell tower is represented by the blue diamond at the point (1.5, 2). Each block represents one mile.  The vehicle travels at a constant rate of  60 mph.   1. In how many minutes will the vehicle be a minimum distance from the cell tower and what is the minimum distance?  Make a conjecture as to when you think the minimum distance will occur. | Note: At this point you may want to have the class work independently or as a group to solve (7) thru (13). As mentioned previously, this would work as a good assessment for the class.  The minimum distance will occur after the car has traveled for 2 minutes: 1.5 miles. |

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| 1. Use the Pythagorean Theorem to find the distance the car is from the cell tower at *t* = 0. | **d = √(1.52 + 22) = 2.5 miles** |
| 1. Complete the table for other values of *t*. | |  |  |  | | --- | --- | --- | |  |  |  | | Time (sec) | Distance Car Travels (miles) | Dist from Cell Tower (miles) | | **0** | **0** | **2.5** | | **30** | **0.5** | **2.121** | | **60** | **1** | **1.803** | | **90** | **1.5** | **1.581** | | **120** | **2** | **1.5** | | **150** | **2.5** | **1.581** | | **180** | **3** | **1.803** | | **210** | **3.5** | **2.121** | | **240** | **4** | **2.5** | |  |  |  | |
| 1. Complete the following:    1. Construct a scatter plot of the data and describe the shape.    2. Consider the distance the vehicle is from the cell tower at *t* = 600, *t* = 1200, *t* = 1800, and *t* = 2400. (Assume the vehicle continues to travel along the same path at the same speed).    3. Describe the data. Did the trend you saw in (a) continue? Explain. | **The answers are similar to those in #4.** |
| 1. You may have noticed that you can find the *y-value* of the point by dividing the time *t* by 60. This means that we can write the coordinates at any times as  (0, *t* / 60). Find the formula for the distance between the point (0, *t* / 60) and (1.5, 2). Solve for ***d***. |  |
| 1. Graph the formula from (11) on the scatter plot from (10).  As an extension, you may have a student find the regression model to see how it compares to the formula from (11) and verify (either with algebra or by estimating) that the minimum of that curve is 90 seconds. |  |
| Take a look back at Figure 1. We’ll now look at another scenario, similar to Figure 1.  A vehicle, represented by the red dot at the point (0, 0) is traveling along a road represented by the line *y = -x* towards  the upper left of the graph. A cell tower is represented by the blue diamond at the point (-2, 4.5). The scale remains one unit equals one mile.  The vehicle travels at a constant rate of 60 mph.  We want to find out the number of minutes until the vehicle will be a minimum distance from the cell tower and what the minimum distance is.  **Students should know what they are looking for but may not know how to find it – especially algebraically. That is good. That little bit of frustration adds to motivation and is a part of problem solving. Ask them questions to get them to explain what they are thinking and what process they might use to find this minimum distance.**  **An appropriate response would be a table of values. But then ask how to find the distance along the line y = -x the car travels.**  **You may need to review the algebra of isosceles right triangles.** | **Note: At this point you may want to have the class work independently or as a group to solve (14) thru (20). As mentioned previously, this would work as a good assessment for the class.**    **Since a = b;**  **a2 + a2  = c2**  **2 a2 = c2**  **a = c/√2**  **This means that if the car travels for one minute along c, for one mile, then the x- and y- component of that distance is .707.** |

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| 1. Make a conjecture as to when you think the minimum distance will occur. | | | **A good guess would be around 5 minutes sine 0.707 \* 5 is about 3.5. That would place the car at the point where a segment drawn perpenciducular to the road and through the cell tower would be located.** |
| 1. Use the Pythagorean Theorem to find the distance the car is from the cell tower at  *t* = 0. | | | **d = √((0-(-2))^2 + (0-4.5)^2) = 4.924 miles** |
| 1. Complete the table for other values of *t*. | | | |  |  |  |  | | --- | --- | --- | --- | | Time | Dist Car Travels x | Dist Car Travels y | Dist from | | (sec) | (miles) | (miles) | Cell Tower | | **0** | **0.000** | **0.000** | **4.924** | | **60** | **-0.707** | **0.707** | **4.007** | | **120** | **-1.414** | **1.414** | **3.141** | | **180** | **-2.121** | **2.121** | **2.382** | | **240** | **-2.828** | **2.828** | **1.866** | | **300** | **-3.536** | **3.536** | **1.813** | | **360** | **-4.243** | **4.243** | **2.257** | | **420** | **-4.950** | **4.950** | **2.984** | | **480** | **-5.657** | **5.657** | **3.835** | | **540** | **-6.364** | **6.364** | **4.745** | | **600** | **-7.071** | **7.071** | **5.686** | | **660** | **-7.778** | **7.778** | **6.643** | | **720** | **-8.485** | **8.485** | **7.612** | | **780** | **-9.192** | **9.192** | **8.588** | | **840** | **-9.899** | **9.899** | **9.569** | |
| 1. Construct a scatter plot of the data. Describe the shape of the curve. At what ***t*** is there a minimum value?   How certain are you that the value you see in the table is the minimum value? | | **The minimum actually occurs a little before five minutes, as can be seen in the graph. Therefore, students should want to decrease the increment for x in the table – to zoom in on the minimum.** | |
| 1. Describe the data. Does this data follow the same trends as in the previous two scenarios? Explain. | | | **Yes. The data follows the same trends as before, rather parabolic in shape until t gets relatively large. For large values of t the curve straightens out.** |
| 1. Find the formula that models the distance between the car and (-2, 4.5) for any time, ***t***. | | |  |
| 1. Graph the formula from (18) on the scatter plot from (16).   **You can now ask several questions to see how deeply students understand this scenario and model. Is the graph a parabola? No.**  **How many functions might be required to model this scenario? Probably at least three; two lines and what seems to be a parabola.** |  | | |
| 1. Find the time at which a minimum occurs.   **Using the table in (15), students should see a minimum occurs between t = 240 and t = 360. They should realize that t = 300 probably isn’t the absolute minimum and see the need to ‘zoom in’ to find the “exact” minimum.**  **A good discussion here is how accurate they need to be. Ask, “Isn’t finding the minimum to the closest second good enough?”**  **A good analogy is in weather data. The high temperatures on the hour are usually what appear in data; however the high temperature for the day may actually occur at a time between the hours.** | | | **Minimum occurs after about 275.77 seconds when the car is at about (-1.77, 1.77).**   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **272** | **-3.206** | **3.206** | **-2** | **4.5** | **1.769** | | **273** | **-3.217** | **3.217** | **-2** | **4.5** | **1.768** | | **274** | **-3.229** | **3.229** | **-2** | **4.5** | **1.768** | | **275** | **-3.241** | **3.241** | **-2** | **4.5** | **1.768** | | **276** | **-3.253** | **3.253** | **-2** | **4.5** | **1.768** | | **277** | **-3.264** | **3.264** | **-2** | **4.5** | **1.768** | | **278** | **-3.276** | **3.276** | **-2** | **4.5** | **1.768** |   **The Excel file contains a template that allows the user to change the initial value of t and the increment. This is a useful template for teachers and students and models the “zoom” feature of the calculator but requires understanding on the part of the user since the user enters the intial value of x and the increment!** |

**The Excel file provides another opportunity to extend the problem with two roads, both at angles.**