The Right Stuff: Appropriate Mathematics for All Students

Promoting materials that engage students in meaningful activities, promote the effective use of technology to support the mathematics, further equip students with stronger problems solving and critical thinking skills, as well as enhance numeracy.

Overview

Students will apply the concepts of

- Modeling. Students will investigate the reasonableness of models to data: linear, quadratic, cubic, and exponential.
- Sine Function. Students will use the sine function as a reasonable model for periodic data.

Supplies and Materials

- 14.1 Student Worksheet
- 14.3 Excel file or 14.4 TI-Nspire ™ file
- Access to technology that can calculate regression equations.

Prerequisite Knowledge

Students must be able

- To construct a scatter plot and find a regression equation.
- Use technology to find the value of a function

Instructional Suggestions

The goal of the lesson is for students to observe that what might be called "good models" (with regard to the R^2 value) might not be appropriate. With additional data, students should see the need for another type of function to model this periodic data.

As students investigate these data, you should find opportunities to discuss the meaning of the value for R^2; the problems with extrapolating, even with very similar values for R^2; and the importance of knowing the context of a set of data, not just the data.

Assessment Ideas

Go to the web and find data on the moon (waxing and waning), on the tides (low and high), or on the daily high temperature in your town. Have students find appropriate models.

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Introduction

A goal of college mathematics is to help students learn to deal with data. People often refer to wanting their decisions to be data-driven, or better, data-informed. Data can help you make good decisions but, even with good data, the answer or direction to take is not always crystal clear. This lesson will deal with data and the limitations that are inherent in data analysis.

Context

Examine the data in Table 1.

Table 1

Day	Time
10	15.55
11	15.53
12	15.53
13	15.52
14	15.48
15	15.48
16	15.45
17	15.43
18	15.40
19	15.38
20	15.37
21	15.35
22	15.32
23	15.28
24	15.27
25	15.23
26	15.20
27	15.17
28	15.13
29	15.12
30	15.08
31	15.05
32	15.02
33	14.98
34	14.95
35	14.90
36	14.87
37	14.83
38	14.80
39	14.77
40	14.72

1. Write at least one observation about the "time" data.

The time is decreasing and seems to be decreasing at an increasing rate.

2. Write a feasible scenario that could fit the data. That is, what kind of "story" might this data be a result of?

This data could be the time it takes a jogger to run a mile over the period of a month (time would be in minutes). It could be the time it took a technician to perform a daily task.

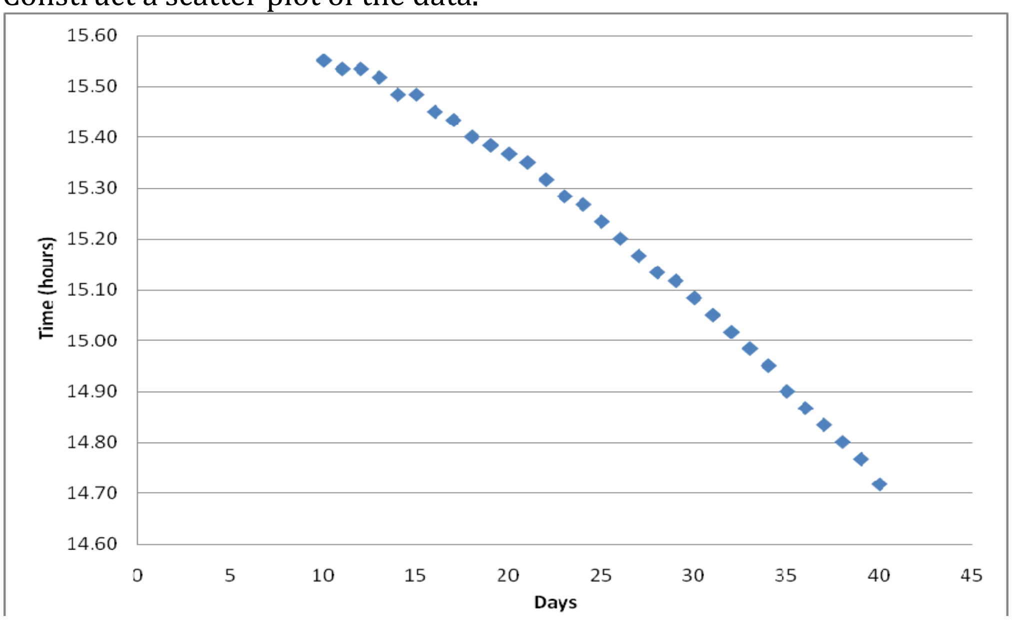
3. Explain why the data does not appear linear.

The data is decreasing at an increasing rate, so no, the data does not appear linear.

4. What type of function would you suggest as a model for this data?

A polynomial, probably of degree 2, or a power function might appear to be good alternatives.

Construct a scatter plot of the data.



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5. Use regression analysis to complete the following table by providing the appropriate equation and r-squared value for each type of function listed. Then, use that equation to predict the value of the dependent variable at the given values of the independent variable.

Type of Model	Equation	r-squared	-30	180
Linear	**-0.02805(time)+15.9**	**0.985**	**16.74**	**10.851**
Quadratic	**$-0.000423(time)^2-0.006879(time)+15.669$**	**.99935**	**15.49**	**0.7256**
Cubic	**$0.000004t^3-0.000708t^2+0.000314t+15.624$**	**.99942**	**14.87**	**16.069**
Exponential	**$15.91584e^{-0.001849(time)}$**	**.98335**	**16.82**	**11.410**

6. The four types of functions give different predictions even though the r-squared values are all very high. What is missing in the exercise thus far? What additional information might provide you with enough information to make a reasonable choice for the most appropriate model?
The scenario would help us determine the type of function that would be most appropriate.

7. Here is the scenario for which the data applies:

> The data is taken from http://www.timeanddate.com/worldclock. Among other things, this site provides data on the time of sunrise and sunset for many locations around the world. The data in Table 1 shows the length of time between sunrise and sunset for Minneapolis, MN, in hours, for the days in the month of July (days after June 21, 2007). The data is rounded to the nearest minute.

Given this scenario and what you know about the equinoxes, why was June 21 a "start date?"
June 21 is the longest day of the year (sunrise to sunset), the summer solstice. The days, for locations north of the equator, get shorter (decreasing at an increasing rate) through the autumnal equinox (September 22) and then continue to decrease at a decreasing rate through December 21, the winter solstice.
***Math Across the Curriculum: Discuss the location of the sun relative to Minneapolis (or your location) during the equinox and solstice.**

8. Explain why none of these functions are likely to be accurate for predictions for day = -30 and day = 90.
Day = -30 means the date is around May 21, on the other side of the solstice. The days do not continue to increase in length past day zero; rather, they begin to decrease in length from day = 0 to day = -30.
Day = 180 is near the winter solstice so the length of the days have reached a minimum and will begin to increase. The function would need to decrease at an increasing rate and then decrease at a decreasing rate before reaching that minimum. The cubic might do that but the value in the table indicates it does not.

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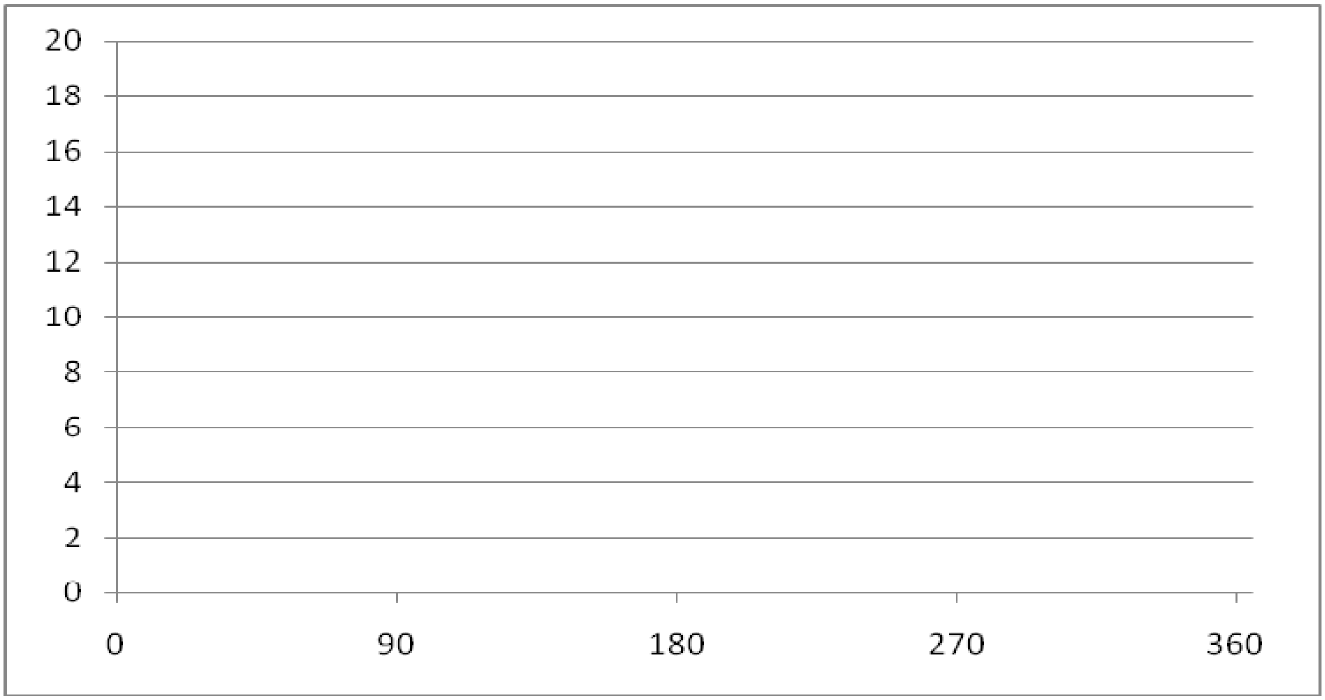
9. The length of day is cyclical. That is, the four seasons produce a curve that has a minimum and a maximum, and between each of those, the curve passes through what might be called a median line. Make a sketch. (Do not use other resources to find exact times; rather, estimate based on what you know.) of the length of the day for your area with day = 0 being June 21, 2009.



Why are the maximums and minimums of your sketch where they are? Explain.
The curve should begin at a maximum because June 21 is the longest day of the year. Students may assume that the average length of a day is 12 hours, but it does depend on the location. The minimum should occur at about 183.
Discuss why the length of the longest day in Minneapolis is not the same as the length of the longest day at your location (assuming your location is not on the same latitude as Minneapolis).

10. The shape of the curve you made in #9 should be very much like a wave. None of the functions listed in the table models this type of behavior. However, each function provided a good model, relative to the r-squared value, for the original data. The four models you found in #5 can be used to approximate the *time* within the range of *days* given; $10 \le day \le 40$. However, as you determined, for values of *day* beyond the given data, the models do not provide good estimates for *time*.

The lesson is "In order to assess the appropriateness of a model, we need to know the scenario." Even then, using a model to predict values of the dependent variable outside the given range of the independent variable is risky.

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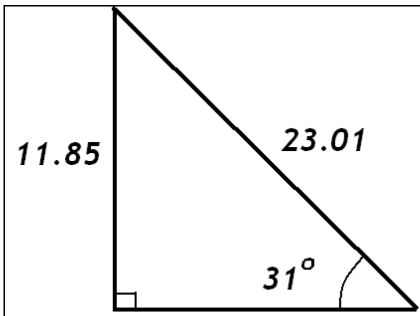
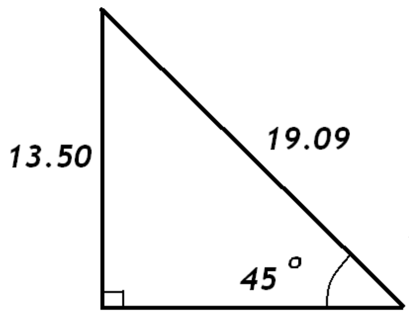
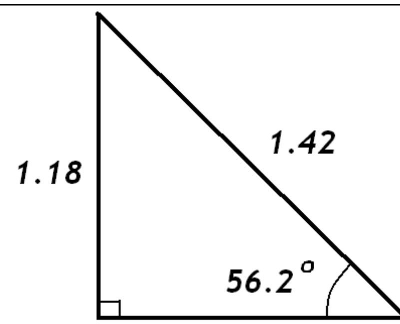
The sine function is a trigonometric function. It can be used to model cyclical data. In this case, the sine function provides the best model.

SINE (INPUT) = OUTPUT
sin (input) = output
sin (45°) = .707
sin (.785398) = .707

The sine function must have an input and then will provide an output. In many applications, the input is the measure of an angle and the output is a ratio; the ratio of the opposite side of a right triangle to the hypotenuse. The input may also be in radians (another way to measure an angle).

Below, you'll find three different right triangles with given information. Show that if the input is the measure of the angle given (in degrees) the output is the ratio of the opposite side to the hypotenuse. (Be sure your calculator is in DEGREE MODE to make these calculations.) If you are using Excel, it will be necessary to change the input from DEGREES to RADIANS. You do this by multiplying the measure of the angle in degrees by π/180°. (Drawings are not to scale.)

		
Sin (31.0) = .515	Sin (45.0) = .707	Sin (56.2) = .831
11.85/23.01 = .515	**13.50/19.09 = .707**	**1.18/1.42 = .831**

11. Below, you'll find one year of data on the length of the day (sunrise to sunset) for Minneapolis, MN for 2009. (June 21, 2009 = Day 0) Compare this curve with the curve you sketched in #9. Why should or shouldn't the two curves be similar? Different?

The two curves should be similar in that they have maximums and minimums at the same *day* values. However, the *time* max and min should be different, depending upon the latitude of the location.

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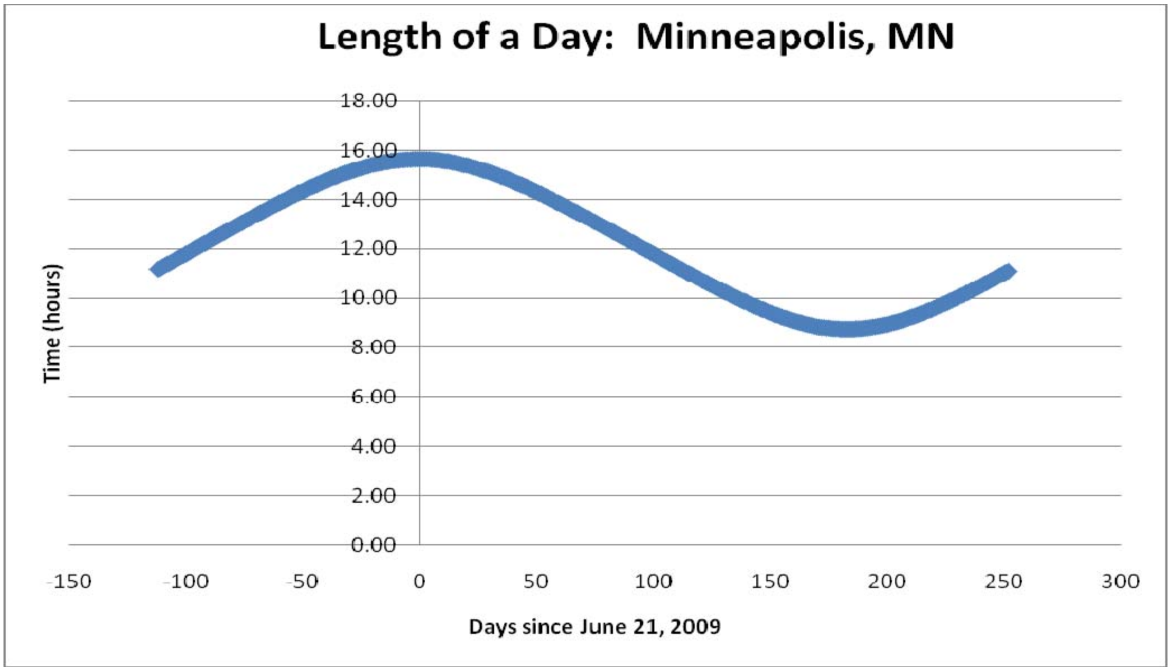

12. To show that the sine function provides a good model for this data, graph either of the following two functions and compare the graph of it to the original data for Minneapolis. Select the one based on whether you are using degrees or radians.

In radian mode, graph: y = 3.42* sin (0.017442 x + 1.58319) + 12.17
In degree mode, graph: y = 3.42* sin (0.999333 x + 90.7099) + 12.17

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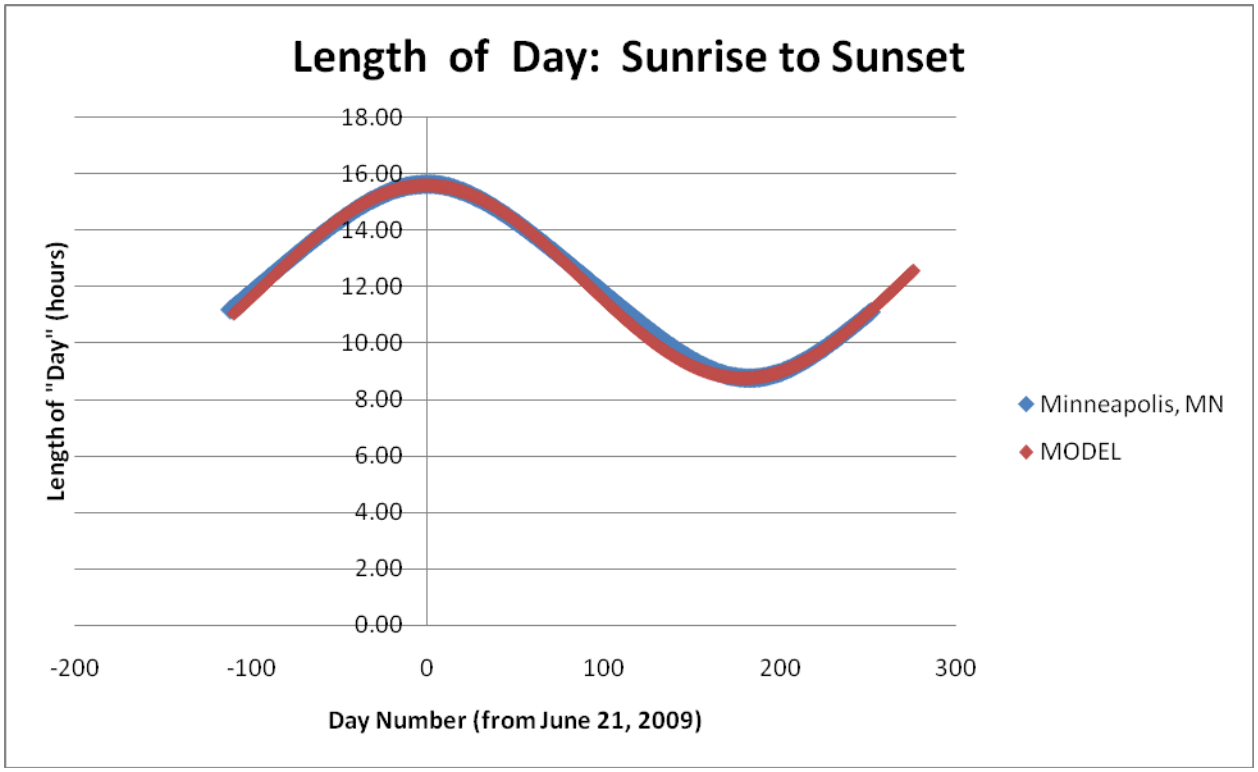
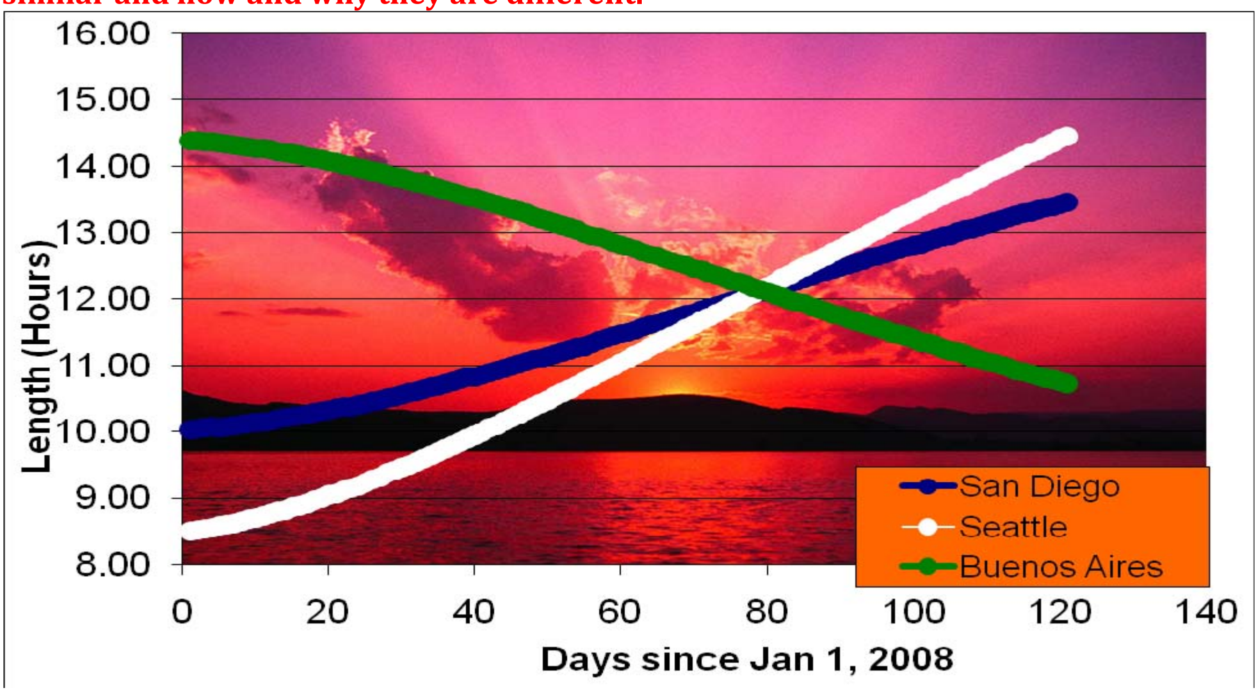


Extension

Compare the graph of the length of day in Seattle with that of San Diego and Buenos Aires (for example). Have students locate these cities and explain how and why the graphs are similar and how and why they are different.



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