The Right Stuff: Appropriate Mathematics for All Students

Promoting materials that engage students in meaningful activities, promote the effective use of technology to support the mathematics, further equip students with stronger problems solving and critical thinking skills, as well as enhance numeracy.

AMATYC
The Right Stuff
Appropriate Mathematics for All Students

Overview
Students will apply the concepts of
- Linear Function – Students will piece linear functions together to make a piecewise defined function.
- Domain and Range – Students will discuss the appropriate range of values for the domain and range in the context of the scenario.

Supplies and Materials
- 15.1 Student Worksheet
- 15.3 Excel file or a handheld that will create a graph

Pre-requisite Knowledge
Students must be able to copy data from Word into Excel or into a handheld, create a scatter plot, and find an appropriate algebraic model for the data. Students must be able to graph linear functions using the slope and intercept.

Instructional Suggestions
Place students into six groups. Have each group work through one scenario asking for help as needed. Once the six groups have completed their work, have the two groups that were working on the same scenario compare their results and prepare a presentation to the rest of the class.

Introduction

There are several functions that may be used to model data: linear, quadratic, cubic, power, etc. However, in some applications, one function, or even one type of function, may not be an appropriate model. The data may require that we piece two or more functions together, over separate intervals, to model the data. This activity will examine data that requires the use of piecewise-defined functions in order to model the data.

Table 1 shows the tuition cost per credit at a local college.

1. Complete Table 2 using the information in Table 1.

Table 1

Number of Credits	How to Determine Your Tuition
1 to 3	$180
4 to 12	$60 per credit hour
13 to 18	$720
More than 18	$40 per credit hour

Table 2

No. of Credits	Total Tuition Cost
1	$ 180.00
2	$ 180.00
3	$ 180.00
4	$ 240.00
5	$ 300.00
6	$ 360.00
7	$ 420.00
8	$ 480.00
9	$ 540.00
10	$ 600.00
11	$ 660.00
12	$ 720.00
13	$ 720.00
14	$ 720.00
15	$ 720.00
16	$ 720.00
17	$ 720.00
18	$ 720.00
19	$ 760.00
20	$ 800.00

2. Graph the data from Table 2 in Figure 1.
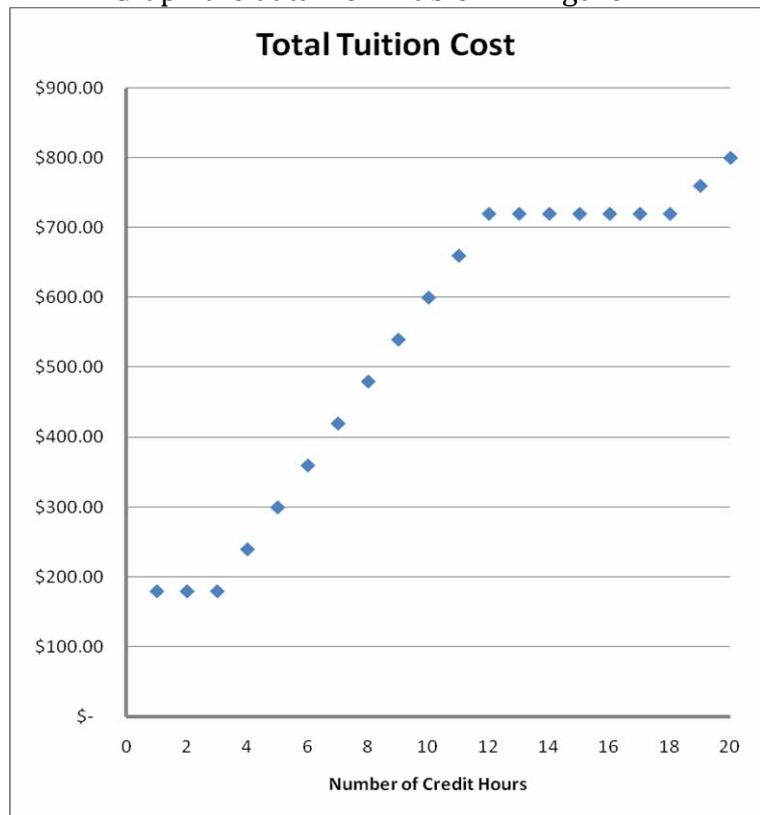
Figure 1

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3. Using complete sentences, describe how the tuition at this college is structured
   In order to calculate the tuition, first look at the number of credits. If the number of credits is less than four, the tuition is $180; between 4 and 12, the tuition is found by multiplying the number of credits by $60. The tuition is $720 when the number of credits is between 13 and 18. The tuition is $40 per credit hour if the number of credits is more than 18.
4. Have someone read your explanation and ask them to determine, from your written explanation, the tuition if a student is taking 2 credit hours, 10 credit hours, or 20 credit hours. Make adjustments to your explanation if necessary.

In order to construct an algebraic model for the tuition, we have to fully understand what calculations to make based on the conditions in the problem. This chart is an intermediate step between the information initially provided and an algebraic model.

Chart 1

$$C(h) = $$

What to do…	*When* to do it…
Pay $180	when number of credit hours is 3 or less
Multiply the number of credit hours by $60	When number of credit hours is from 4 to 12
Pay $720	when number of credit hours is from 13 to 18
Multiply the number of credit hours by $40	when number of credit hours is more than 18

5. Complete Chart 2 by writing the expressions in Chart 1 in the language of Algebra.

Chart 2

$$C(h) = $$

What to do…	*When* to do it…
= $180	$h \leq 3$
= $60 x h	**$4 \leq h \leq 12$**
= $720	**$13 \leq h \leq 18$**
= $40 x h	**$h > 18$**

The algebraic model is one way to represent a piecewise defined function. They can also be represented using a table or a graph.

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The town of Cary, NC uses a piecewise function to determine the monthly water charge for a consumer. Let W represent the cost of water and g the number of gallons, in thousands, used by the consumer. The model for the monthly water bill is

$$W(g) = \begin{cases} 3.28g + 2.76 & 0 \le g \le 5 \\ 3.75(g-5) + 19.16 & 5 < g \le 8 \\ 5.33(g-8) + 30.41 & g > 8 \end{cases}$$

6. Find W(3.5) and explain what these values represent.
    W(3.5) = $14.24. This means that a customer who used 3500 gallons of water would receive a water bill of $14.24.

7. A consumer used 8200 gallons of water in one month. Use functional notation to represent the cost of water for that consumer.
    W(8.2) = 5.33(8.2-8)+30.41 = $31.48

8. How many gallons of water were used if the customer's water bill was $24.79?
    W(g) = $24.79 = 3.75(g – 5) + 19.16
    3.75(g – 5) = 5.63
    g – 5 =1.50133
    g = 6.5 or 6500 gallons of water (rounded to nearest 100 gallons)

9. Sketch the graph of W(g). Describe what the result would be.
    The graph of W(g) contains three line segments, each with a different slope. The end of one segment is the beginning of the next segment.

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Piecewise-Defined Functions

New legislation has changed the rates for computing individual income tax on taxable income. Table 3 is to appear in the new tax guide.

Table 3

If your taxable income (Line 42) is	then compute your income tax using this formula:
less than $25,000	Tax = .15 x (Line 42)
between $25,000 and $60,000	Tax = .18 x (Line 42 - $25,000) + **A**
over $60,000	Tax = .225 x (Line 42 - $60,000) + **B**

However the values for **A** and **B** must first be computed.

10. Find the value of **A** so that a person who has a taxable income of $25,000 can use either of the first two rules and still owe the same amount.
$3750

11. Find the value of **B** so that a person who has a taxable income of $60,000 can use either of the last two rules and still owe the same amount.
$10,050

A customer may choose between two cell phone plans; Plan A and Plan B.
Plan A: $40 for the first 300 minutes and then $.40 per minute for every minute over 300.
Plan B: $25 for the first 200 minutes and then $.30 per minute for every minute over 200.

12. Find the number of minutes for which Plan B is cheaper than Plan A.
Either use a table or graph to verify the answer you give.

Plan B is cheaper if the number of minutes used is between 250 and 450.