The Right Stuff: Appropriate Mathematics for All Students

Promoting the use of materials that engage students in meaningful activities that promote the effective use of technology to support mathematics, further equip students with stronger problem solving and critical thinking skills, and enhance numeracy.

Overview

Students will apply the concepts of

- Formulas – The student can calculate the output of a function given an input.
- Table of Values – The student can create a table of values for a function in two variables.
- Non-linear Functions – The student can recognize that a table of data represents an increasing change in y with respect to x and determine that a non-linear function, specifically the exponential or power function, is the appropriate model for that data.

Supplies and Materials

- 17.1 Student Worksheet
- Either 17.3 Excel or 17.4 TI-Nspire ™ or a handheld that will create a scatter plot and find a model for the data

Prerequisite Knowledge

Students must be able copy data from Word into Excel or into a handheld, create a scatter plot, and find an appropriate model for the data.

Instructional Suggestions

1. Have students calculate the future value (dependent variable) using the rate, Present Value, and number of compounding per year as constants with the number of years as the independent variable.
2. Discuss the rate of growth in the future value as the term changes incrementally. This discussion, along with the table and graph, should clearly show that this is an exponential relationship.
3. Allow students to investigate the relationship between FV and t (holding r, PV, and n constant) using the spreadsheet model.

Modules 16 and 17 use the compound interest formula. Module 16 examines the linear relationship between the PV and FV while Module 17 examines the relationship between t and FV as well as the relationship between r and FV.

Assessment Ideas

1. The formula used to compute the future value of a deposit that earns interest compounded periodically is $\mathbf{FV = PV\ (1 + r/n)^{nt}}$ where FV = Future Value, PV = Present Value, r = Annual Interest Rate, n = Number of Compounding periods per Year, and t = Number of Years

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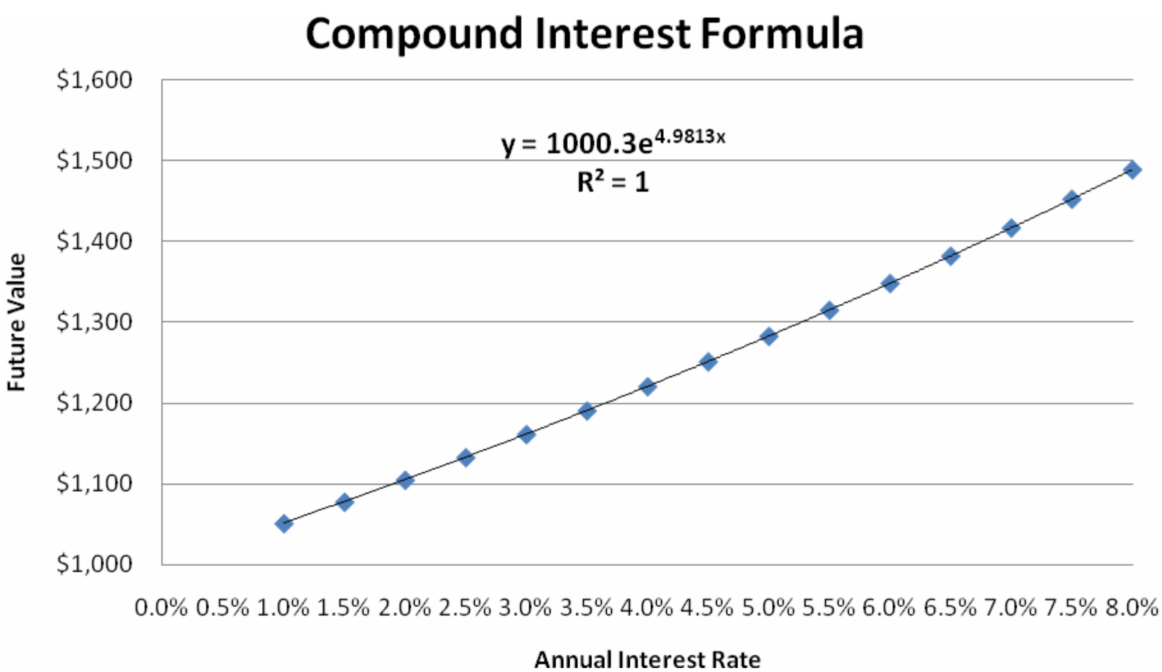
Complete the table using the formula and technology, to find the future value of an initial deposit of $1,000 that earns interest compounded monthly at the given interest rate for five years.

A P R (rate)	Future Value
1.00%	**$ 1,051.25**
1.50%	**$ 1,077.83**
2.00%	**$ 1,105.08**
2.50%	**$ 1,133.00**
3.00%	**$ 1,161.62**
3.50%	**$ 1,190.94**
4.00%	**$ 1,221.00**
4.50%	**$ 1,251.80**
5.00%	**$ 1,283.36**
5.50%	**$ 1,315.70**
6.00%	**$ 1,348.85**
6.50%	**$ 1,382.82**
7.00%	**$ 1,417.63**
7.50%	**$ 1,453.29**
8.00%	**$ 1,489.85**

2. Create a scatter plot and determine an appropriate model for the data and describe general trends in the data set.



The FV is increasing at a decreasing rate.

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Introduction:

Albert Einstein was asked, "What is the greatest invention of mankind?" His reply, "Compound interest."

There are different types of savings accounts that incur interest that compounds periodically. Certificates of Deposit (CDs) and Treasury Bills are two of those different types of savings.

For ease of arithmetic, suppose you earn $50 a year as interest on a $100 deposit. If you only earn interest on the $100 principal investment, you would have earned $250 at the end of three years.

Bank deposits earn compound interest. Suppose you earn 50% of the deposited amount including previously earned interest.
The graph illustrates the effect of adding earned interest to the principal before calculating that year's interest amount. If you only earn interest compounded annually on the $100 principal investment, you would have earned $337.50 at the end of three years.

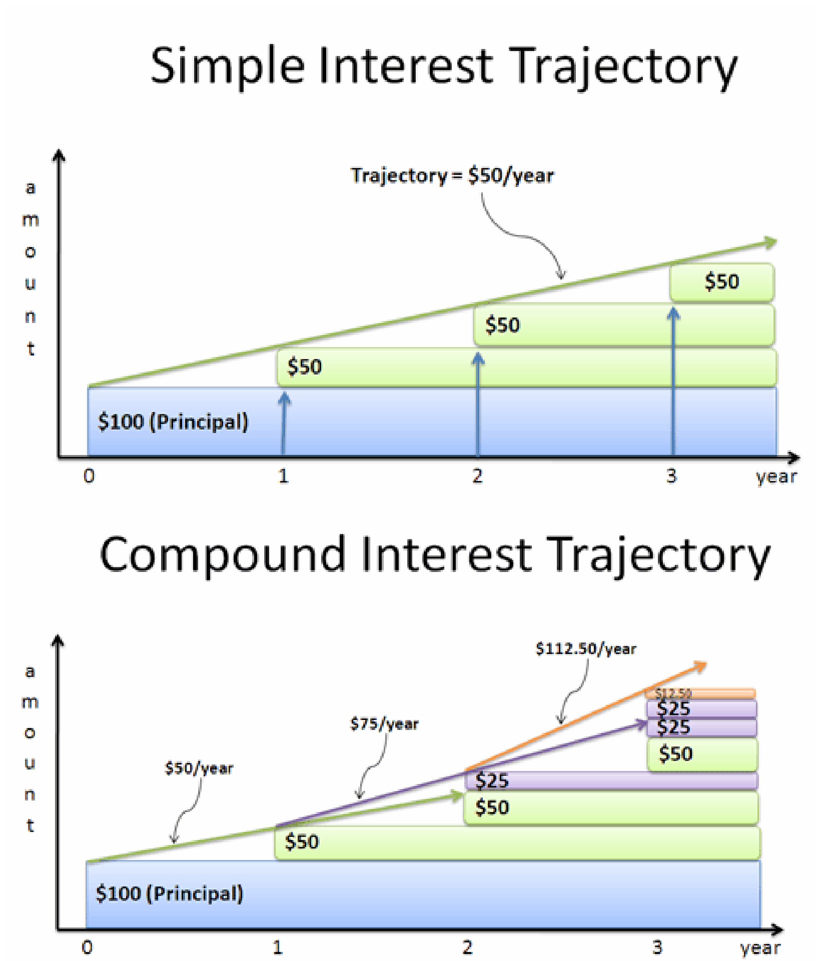

Figure 1
http://betterexplained.com/articles/a-visual-guide-to-simple-compound-and-continuous-interest-rates/

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The formula used to compute the future value of a deposit that has interest compounded periodically is

$$\mathbf{FV = PV\,(1 + r/n)^{nt}}$$

FV = Future Value
PV = Present Value
r = Annual Interest Rate
n = Number of compounding periods per year
t = Number of Years

In the next few steps, you will complete the table (right).
Given:

PV = $1000

r = 6%

n = 12

1. Find the future value of an initial deposit of $1,000 if it earns interest at 6% compounded monthly.

2. Find the Annual Increase in the FV. Describe its rate of increase.

   The FV is increasing at an increasing rate.

   What do the numbers in this third column mean in the context of this problem?

As the term of the loan is longer, there is a larger annual increase in future value.

3. Find the Annual Percent Change in the FV.

Term of Loan (Years)	Future Value	Annual Increase In FV	Annual Percent Change
0	$ 1,000.00		
1	$ 1,061.68	$ 61.68	6.17%
2	**$ 1,127.16**	**$ 65.48**	**6.17%**
3	**$ 1,196.68**	**$ 69.52**	**6.17%**
4	**$ 1,270.49**	**$ 73.81**	**6.17%**
5	**$ 1,348.85**	**$ 78.36**	**6.17%**
6	**$ 1,432.04**	**$ 83.19**	**6.17%**
7	**$ 1,520.37**	**$ 88.33**	**6.17%**
8	**$ 1,614.14**	**$ 93.77**	**6.17%**
9	**$ 1,713.70**	**$ 99.56**	**6.17%**
10	**$ 1,819.40**	**$ 105.70**	**6.17%**

4. Based on your responses to the previous questions, what function should be used to model the (term, FV) data?

An exponential function, by definition, has a constant rate of change from one term to the next. The percent change, 6.17% is the annual yield of a 6% APR.

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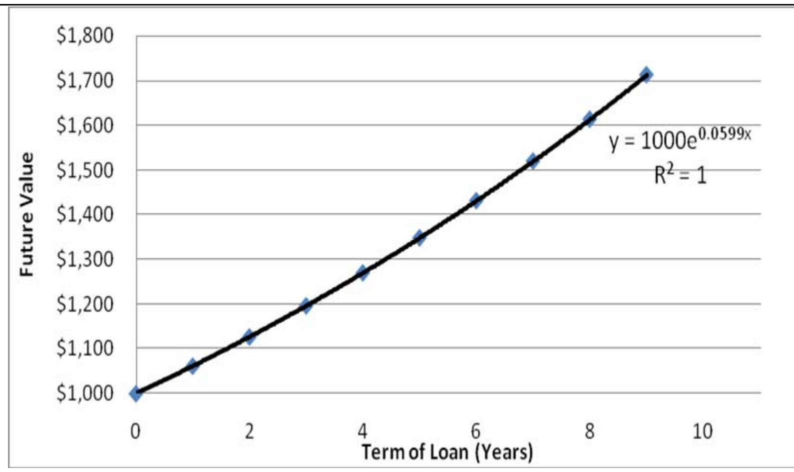
5. Construct a scatter plot and find the function that best models the data. **Using Excel, the function that models the curve is** **FV = 1000 $e^{.0599term}$** **Using exponential regression on a handheld, FV = 1000 (1.06167^{term})** **= 1000 $(1 + 0.06167)^{term}$** **is an equivalent exponential equation. A cubic equation also produces an R-squared of 1. Discuss why the exponential is more appropriate. (The exponential function is typically used for growth and the constants in the exponential model are meaningful.)**	
6. What will happen to the data, (term, FV), if the interest rate changes? If it increases? If it decreases?	**Use a slider to adjust the interest rate and watch the function (see 17.3 Excel). The difference in the FV expands as the interest rate increases.**
7. The exponential function with base e found in (5) can be rewritten with an equivalent exponential function using the value of $e^{.0599}$. Use that value to rewrite the exponential model.	**$e^{.0599}$ = 1.06173** **FV = 1000 * 1.06173^{term}**
8. The value 1.06173 tells us the effective yield of 6% compounded monthly is 6.173%. Explain why this rate is higher than 6%.	**The annual rate is the rate used to compute the interest each period. Every month, interest is added to the principal on which the interest is calculated the next month. Thus, the more compoundings the more interest. Interest compounded monthly has a higher effective yield than does interest compounded quarterly or annually.**
9. Find the effective yield of 4.5% compounded quarterly.	**The effective yield of 4.5% compounded quarterly is 4.64%.**

Extension:
Introduce the Effective Annual Interest Rate formula
Effective annual interest rate = $(1 + R / P)^n - 1$
Where R is the nominal rate and n is the number of compounding periods.

For example, let's assume you have a loan with 8.25% nominal interest. When you make monthly payments (12 compounding periods), your effective annual interest rate will be:
$(1 + 0.0825 / 12)^{12}$ - 1 = 8.57%

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