The Right Stuff: Appropriate Mathematics for All Students

Promoting the use of materials that engage students in meaningful activities that promote the effective use of technology to support mathematics, further equip students with stronger problem solving and critical thinking skills, and enhance numeracy.

Overview

Students will apply the concepts of

- Modeling – Students will be able to find an appropriate model between two variables using technology
- Multiple Representations – Students will be able to represent the relationship between two variables in a table or graph

Supplies and Materials

- 18.1 Student Worksheet
- 18.3 Excel file (available solutions for the instructor)

Prerequisite Knowledge

Students must be able to enter a list of data into a spreadsheet or handheld. Using the list of data, students will have to create a scatter plot, regression line, and graphs of equations.

Instructional Suggestions

1. Discuss the relationship of weighing several different objects and then re-weighing them once they are submerged in water.
2. Have students do the algebra to solve several equations given a value for the radius of a sphere.

Assessment Ideas

Have the students compare the weights of objects with the weight of the object once it is submerged in a substance other then water. How does it compare to the ratio that was found when objects were submerged in water.

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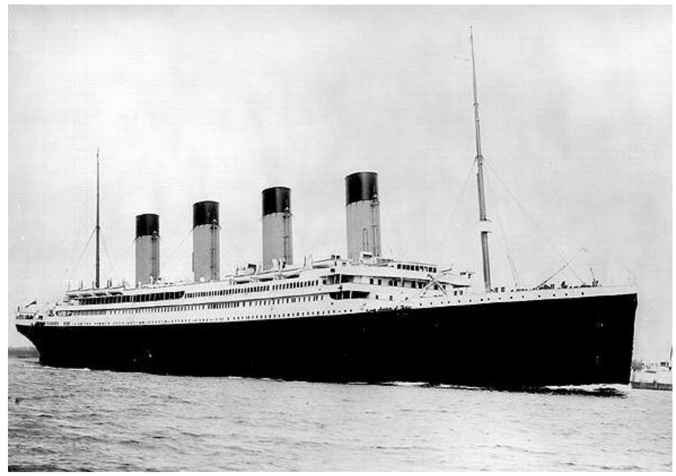
Introduction

The most famous cruise ship of all time is probably the Titanic. The Titanic was 882' 9" long and 92' 6" wide. Filled with 2,228 passengers and crew, it weighed about 46,000 tons. Have you ever considered how such a huge vessel remains afloat?

Well, obviously it didn't! However, without a hole in its hull, it could still be sailing across the vast ocean.

This activity will investigate the property that explains the force that keeps objects like the Titanic afloat.



Data Collection

Setup: Fill a fairly large beaker with water. Find several metal objects (cubes or spheres), some string, and a spring balance.

Calculate the volume of each object that will be submerged.

Attach one of the objects to the spring scale. (See Figure 1.) Record the weight of the object in air. Submerge the object but do not allow it to touch the walls of the container. Record the apparent weight of the object while submerged. Find the difference in the two weights.

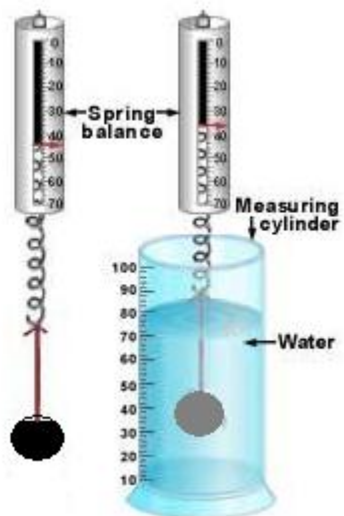


Figure 1

Attach another object to the spring scale and record the weight of the object in air and then in water. Repeat this process to collect at least five sets of data. (Note: if similar objects are used, like steel balls, more than one steel ball can be tied to the string at one time.) Sample data is shown in Figure 2.

Construct a scatter plot of the data and find the equation of the line of best fit.

Figure 2

D6 =B6-C6

	A	B	C	D
1	Volume	Weight $_{AIR}$	Weight $_{WATER}$	ΔW
2	31.6	270	240	30
3	27.4	250	225	25
4	33.5	285	250	35
5	44.5	325	280	45
6	19.5	200	180	20

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Results

Depending upon the accuracy of the data, the scatter plot should demonstrate that the rate of change of weight difference (ΔW) relative to the volume of the object is 1 gram per cubic centimeter. See Figure 3.

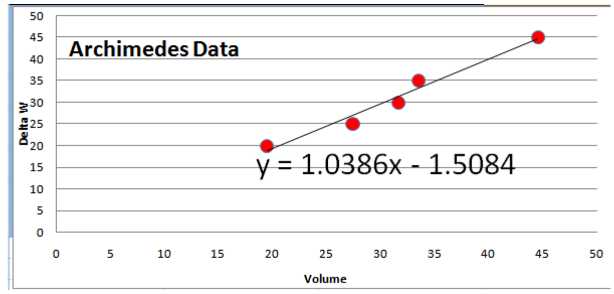

Figure 3

Archimedes' Law

An object submerged in water weighs less than it does out of water because of the buoyant force exerted on it by the liquid. Archimedes' Law states that a body immersed in a liquid, wholly or partly, loses its weight. The loss of weight is equal to the weight of the liquid displaced by the body. Water has a density of 1 gram/cm^3. In English units, that is 62.4 pounds per cubic foot.

Thus, if a body displaces an amount of water exceeding its own weight, it will float. An object that weighs more than the weight of the water it displaces will sink. See Figure 4.

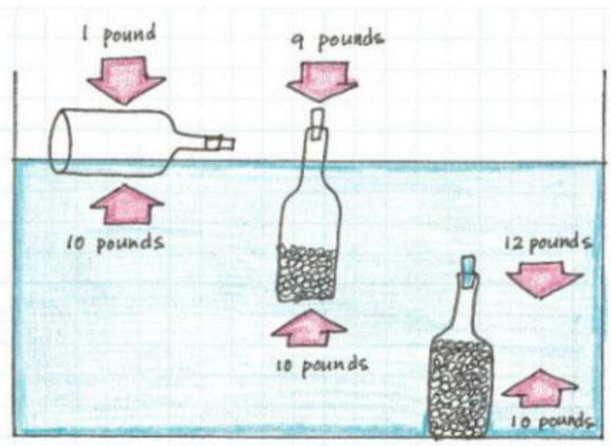

Figure 4

Spherical Cap

The portion of a sphere cut by a plane forms a spherical cap. See Figure 5. If the radius of the sphere is $\boldsymbol{r}$, the radius of the base of the cap is $\boldsymbol{a}$, and the height of the cap is $\boldsymbol{h}$, then it can be shown that

$$V_{cap} = \pi h(3a^2 + h^2)/6 \quad \text{(equation 1)}$$
$$a^2 = h(2r - h) \quad \text{(equation 2)}$$

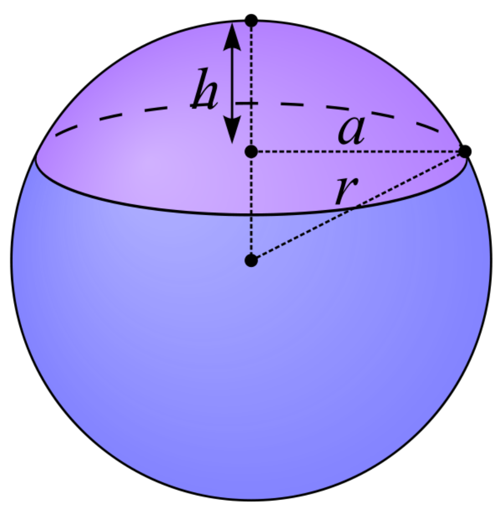

Figure 5

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The water level of a ball floating in water is determined by several factors; predominately the density of the ball and the density of the liquid in which it is floating.

Archimedes' Law states that the volume of the ball under water will displace the amount of water equal to the weight of the ball.

The volume of a ball underwater is a spherical cap and must displace the exact amount of water equal to the weight of the ball. See Figure 6.

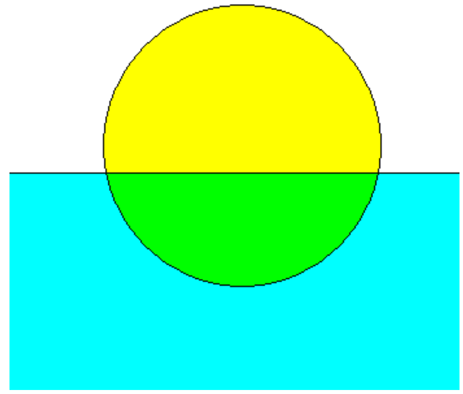
Figure 6

Problem

A wooden ball of diameter 4 feet weighs 600 pounds. The ball is floating in water. Determine the depth to which the ball is submerged.

Solution

1. Identify the value ***r*** as used in equation 1 and equation 2.
   r = 2 feet

2. Simplify equation 2 by substituting the value of r.
   $a^2 = h(2r - h) \rightarrow a^2 = h(2(2) - h) \rightarrow a^2 = h(4 - h)$

3. Substitute the right-hand side of the simplified version of equation 2 for a^2 in equation 1 and simplify.
   $V_{cap} = \pi h(3a^2 + h^2)/6 \rightarrow V_{cap} = \pi h(3h(4 - h) + h^2)/6 \rightarrow V_{cap} = \pi h^2(2 - h/3)$

4. The units for the volume of the cap are in cubic feet as long as the distance ***h*** is measured in feet. Since water has a density of 62.4 pounds per cubic feet, the result of multiplying the volume of the region by the density of the liquid being displaced (water) will result in the weight of the displaced water. Write the Weight of the displaced water, W, as a function of the depth the ball is submerged, ***h.***
   $W_{water}(h) = 62.4\,\pi h^2(2 - h/3)$

5. Since the ball weighs 600 pounds, W_{ball} = 600 pounds. Find the solution to the equation $W_{water}(h) = W_{ball}$.

   (a) Construct a table for h = 0 to d = 4 in increments of .25 feet showing the value of $W_{water}(h)$ as well as the value of W_{ball}. Identify between which two values of d the solution may be found.
   (b) Graph the data in part (a) to show the solution to the equation.

6. Explain the meaning of the solution in terms of the floating ball.

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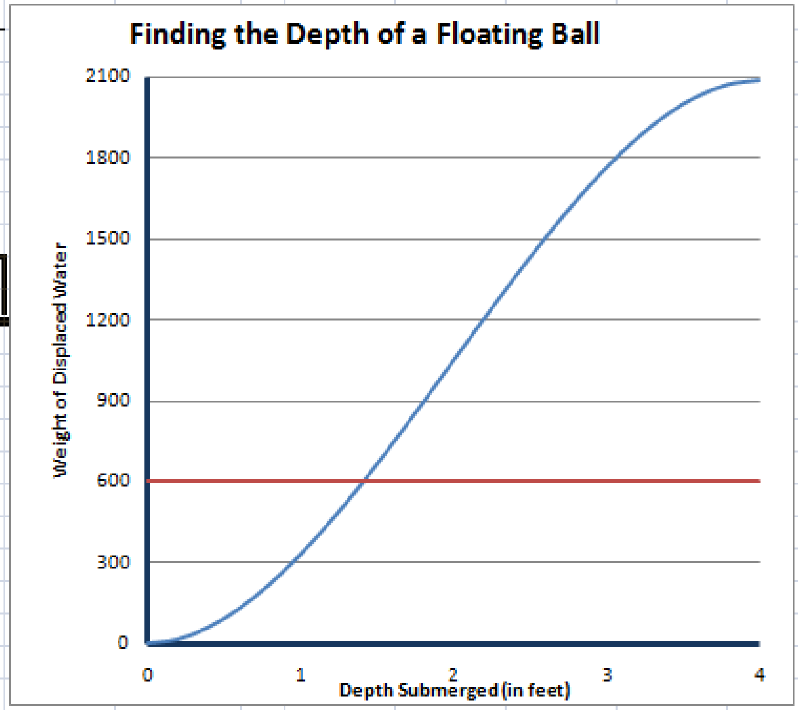
	depth (in feet)	W_{water}(depth)	W_{ball}
2	0	0	600
3	0.2	15.16006951	600
4	0.4	58.54923397	600
5	0.6	127.0309273	600
6	0.8	217.4685833	600
7	1	326.725636	600
8	1.2	451.6655192	600
9	1.4	589.1516668	600
10	1.6	736.0475127	600
11	1.8	889.2164909	600
12	2	1045.522035	600
13	2.2	1201.827579	600
14	2.4	1354.996558	600
15	2.6	1501.892403	600
16	2.8	1639.378551	600
17	3	1764.318434	600
18	3.2	1873.575487	600
19	3.4	1964.013143	600
20	3.6	2032.494836	600
21	3.8	2075.884001	600
22	4	2091.04407	600



24	d is depth of water			
25	$62.4\pi d^2(2-d/3) = 124.8\,\pi\, d^2 - 20.8\,\pi\, d^3$		1.4151	599.953
26	$600 = 124.8\,\pi\, d^2 - 20.8\,\pi\, d^3$	equilibrium depth is d =	1.4152	600.025
27	$0 = 20.8\,\pi\, d^3 - 124.8\,\pi\, d^2 + 600$		1.4153	600.097
31–32	Density of the wooden ball is	weight/volume =	600 pounds / 33.51032 cubic feet	
33	17.9049311 pounds per cubic foot - a VERY light wood.			

The 600-pound ball will be submerged to a depth of 1.4152 feet or about 17 inches.

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