Introduction to the Activity

Maybe you've left a hot cup of coffee on your desk and then got involved in an email and forgot it was there. Minutes later, you think "coffee" and take a sip. It is too cool and needs warming.

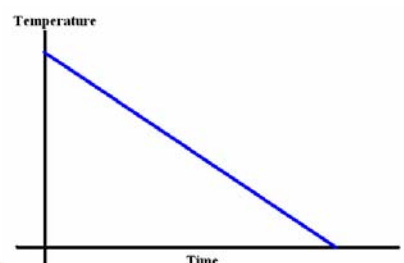
You might not consider it, but what you've witnessed is an application of physics. Isaac Newton himself thought about it, he just didn't have a microwave to remedy the problem of a cold cup of coffee.

Consider a hot cup of coffee, or water, placed on a desk. List up to four factors you believe will influence the rate at which the coffee cools.

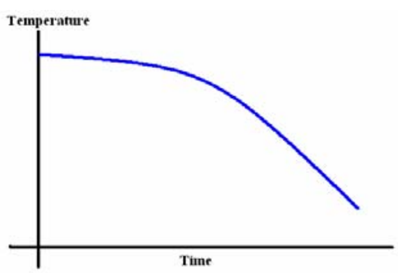
1. The temperature of the surrounding room.
2. The amount of surface area exposed to the environment
3. The amount of wind flow over the surface area of the coffee.
4. The material that is holding the coffee – the degree to which it insulates
5. The amount of mixing that takes place during cooling
6. The humidity of the environment within which the coffee is placed

After making your list, discuss your list with another student. Make changes if you want, but note why you made the change. That is, what your partner said to cause you to make the change.

Finally, with your partner, select the one graph below that you believe represents a good model for the temperature of the cooling liquid over time.



a



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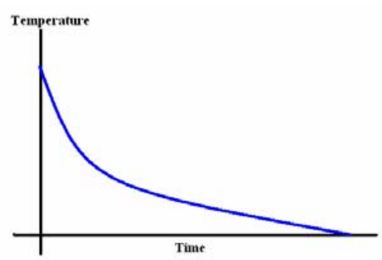
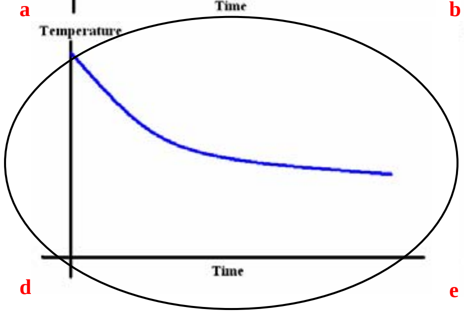


c



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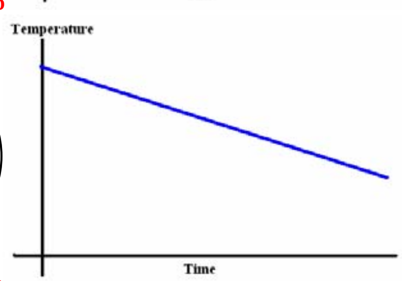

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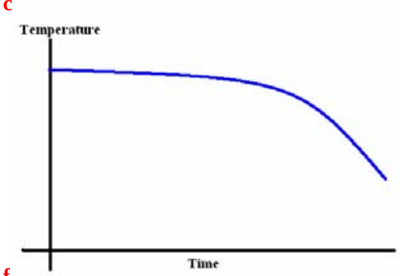

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Module 5

Newton's Law of Cooling – An Application of Exponentials and Logarithms

Task:

Find an appropriate model for the temperature of a cooling liquid over time.

Given Information:

Given a "hot" liquid, Newton's Law of Cooling states that the rate at which the liquid cools is proportional to the difference in the temperature between the liquid and the room temperature (or *ambient temperature*, T_α). This relationship is modeled by the formula:

$$T_t = a + be^{kt}$$

The values of *a, b* and *k* are affected by variables in the experiment.
What variables? What will change the rate at which the liquid cools?
These are questions that go beyond a cup of coffee left on a desk! Consider a winter day. When the temperature drops to zero, you want to know what you can do to keep the inside of your house warm.

The class will perform three experiments that will help determine at least one of the factors that can change the rate of cooling.

Group Work

Step 1

Read the directions in Step 2. Discuss the experiment within the group.

Q1. What factors will influence the cooling process and affect the values of *a, b* and *k*?

The difference between the ambient temperature and the liquid, the type of liquid, the type of container (material and whether it is open or closed), and the wind or air flow along the exposed surface of the liquid.

Step 2

The class will be divided into groups. Each group has a temperature probe, a TI-Nspire ® handheld, and a cup of hot liquid. Each group will also have either a porcelain coffee cup, a Styrofoam coffee cup, or a Styrofoam coffee cup with a lid.

Record the room temperature (also called ambient temperature, T_α, pronounced "T sub alpha"). Heat the liquid to a boil and pour it carefully into the appropriate container. Connect the temperature probe to the handheld and turn the handheld on. The temperature probe will show the current temperature of the liquid. To begin automatic data collection (for three minutes – measurements every second), press "MENU", then "1: Actions", and then "9: Data Collection". Once the temperature display has hit a maximum, move the cursor to the right arrow in the temperature display and press the round white button. Data collection will then begin in the spreadsheet. Record the actual time that this begins. <You may want to practice this procedure at least once before using the hot liquid.>

The graph and data are automatically displayed on the handheld.

Record:	Room Temperature (T_α):	__20° C__
	Time:	__9:00 a.m.__
	Initial Temperature (T_0):	__97.1245° C __
	Ending Temperature (T_{180}):	__83.3120° C __

One person in the group will remain with the liquid and take temperature measurements at the ten, twenty, and thirty minute marks.

Record: Temperature (T_{600}): __58° C __

Temperature (T_{1200}): __35° C __

Temperature (T_{1800}): __21° C __

Step 3

We know the model should be $T_t = a + be^{kt}$.

Q2. Should k be greater than or less than zero? Why?

The value of **k** should be less than zero so that the value of $a + be^{kt}$ approaches a.

Q3. What value should a have?

Hint: Over a long period of time, maximum cooling will have taken place. The value of be^{kt} would be negligible.

The value of **a** will be the ambient temperature, room temperature.

Q4. What value should b have?

Hint: Let $t = 0$.

The value of **b** will be the difference between the initial temperature of the liquid and the ambient temperature.

Q5. Compute the value of k .

Hint: Use the temperature at $t = 180$.

Write the model for the data here using the values you found above:

$$83.3120 = 20 + 77.1245\, e^{180k}$$

$$k = -.001096$$

$$\mathbf{T_t = 20 + 77.1245e^{-.001096t}}$$

Step 4

Use the statistical package in TI-Nspire to find a model for the data.

Write it here: $\mathbf{y=96.4874(.999157^{x})}$

Step 5

Copy the data from the handheld into an Excel spreadsheet using the computer link cable.

Use the tools in Excel to find an exponential model for the data.

Hint: After copying the data into Excel, edit the data by removing the constant a. Find the model from the revised data. Don't forget to put that constant back into the model though.

Write it here: $\mathbf{y = 20 + 76.535545e^{-0.001086x}}$

Q6. Compare the models. Are they equivalent? What reasons would cause these two models to be different?

The models from Step 3 and Step 5 are similar algebraically and roughly equivalent numerically. Since the model in Step 3 was found only using the point T_{180}, we would assume the model from Step 5 to be more accurate. The model in Step 4 is algebraically equivalent to the model in Step 5:

$$y = 96.4874(e^{-.00084336\,x}) \qquad y = 20 + 76.4874(e^{k\,x})$$

$$96.4874(e^{-.00084336\,x}) = 20 + 76.4874(e^{k\,x}) \qquad \text{Let } x = 1:$$

$$96.4874(e^{-.00084336}) = 20 + 76.4874(e^{k})$$

$$76.40606 = 76.4874(e^{k})$$

$$.998937 = e^{k}$$

$$k = -.001064$$

The difference in the value of k is due to the number of decimal places in the original numbers. After more than ten minutes the difference in the prediction using k=-.001064 or k=-001086 is only about one-half a degree.

Step 6

Use a model to predict the temperature of the liquid at the ten, twenty, and thirty minute marks. Compare those predictions with the actual temperature. Discuss the results compared to the observed values.

	Predictions
600	**58.18414**
1200	**35.08637**
1800	**21.15789**

The observed values are close to the predictions.

Step 7

Other groups in the class used different containers (porcelain cup, Styrofoam cup, Styrofoam cup with a lid…).

Q7. How do the models compare? What component of the model changes as the container changes and why?

Since the initial temperatures were slightly different, the value of b changed the model somewhat. However, the value of k was also different. The value of k, the rate of cooling, was smallest for the covered container, larger for the Styrofoam cup, and largest for the porcelain cup. This is due to the insulation around the liquid and, to the exposed surface area to the outside temperature.

Q8. What other factors could be introduced to change the model?

If there were wind moving air across the exposed surface area, we would expect a greater rate of cooling.

Conclusion

Fill in the blanks with appropriate words or phrases.

The temperature of a an object placed in an atmosphere of a different temperature will change at a rate proportional to the difference between the temperature of the object and the temperature of the atmosphere. The rate at which the temperature changes can be controlled by the type of container in which the object is placed. The more the object is exposed, the larger the rate will be.
The function that best models this phenomenon is exponential. The exponent is negative if the temperature of the object is decreasing and positive if the temperature of the object is increasing. As t is large, the model will always level out at the ambient temperature, if conditions remain the same.